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CLIL key terms

Term	explanation
Additional language	Increasingly used – replaces second, foreign, minority language
Discourse type	Type of communication- 1 interactional- social, sharing communication 2 transactional- giving and receiving information
Exposure	Amount of time spent on ELT or CLIL Low=5%-15% Medium= 15%-50% High= over 50%
Immersion	Extensive exposure – usually 85%+
Language awareness	Understanding how languages work
Language shower	Frequent but short exposure to CLIL
Learner styles	Learners preferences for ways to learn
Learner strategies	Techniques pupils use to help understanding – must be shown by teacher/peers
Majority language	Language of immediate social environment- sometimes, not always, national language
Monolingual environment	One language widely used in social setting
Mother tongue	Also- 'home language' or 'heritage language' or 'first language'
Plurilingual attitudes	Supporting the use of three or more languages in same environment
Target language	Also 'L2'- other language used for CLIL which is not host language
Translanguaging	When one or more languages used in CLIL classroom
Codeswitching	Personal speech includes words, phrases in more than one language- languages usually shared with interlocutor
Chunk	Two or more words frequently used together and have meaning
Other terms you meet during the course:	

Session 1 Introduction to CLIL Teaching

Diana Hicks

To start you off.....

Monolingual/CLIL pedagogy: Your thoughts..

	I agree	Maybe/ depends	I don't agree
1 All students are able to study curriculum subjects using CLIL	X	~~X~~	
2 CLIL teaching needs a different approach from monolingual teaching	X		
3 We can use all the same tasks in CLIL teaching as we can in monolingual teaching			X
4 The curriculum will move more slowly in CLIL teaching	~~X~~		X
5 Teachers should not use mother tongue in CLIL classes must use Relien language		X	
6 Students should not use mother tongue in CLIL classes must		X	
7 All the reading texts should be in English			X
8 All students written work should be done in English			X
9 In CLIL teaching the curriculum content is less important than language work is more important			X

} Teachers and students must use Tongue Chl.

} books in both language

Can you add any other 'True' sentences of your own?

EUROPEAN LEVELS - SELF ASSESSMENT GRID

		A1	A2	B1	B2	C1	C2
UNDERSTANDING	Listening	I can understand familiar words and very basic phrases concerning myself, my family and immediate concrete surroundings when people speak slowly and clearly.	I can understand phrases and the highest frequency vocabulary related to areas of most immediate personal relevance (e.g. very basic personal and family information, shopping, local area, employment). I can catch the main point in short, clear, simple messages and announcements.	I can understand the main points of clear standard speech on familiar matters regularly encountered in work, school, leisure, etc. I can understand the main point of many radio or TV programmes on current affairs or topics of personal or professional interest when the delivery is relatively slow and clear.	I can understand extended speech and lectures and follow even complex lines of argument provided the topic is reasonably familiar. I can understand most TV news and current affairs programmes. I can understand the majority of films in standard dialect.	I can understand extended speech even when it is not clearly structured and when relationships are only implied and not signalled explicitly. I can understand television programmes and films without too much effort.	I have no difficulty in understanding any kind of spoken language, whether live or broadcast, even when delivered at fast native speed, provided. I have some time to get familiar with the accent.
	Reading	I can understand familiar names, words and very simple sentences, for example on notices and posters or in catalogues.	I can read very short, simple texts. I can find specific, predictable information in simple everyday material such as advertisements, prospectuses, menus and timetables and I can understand short simple personal letters.	I can understand texts that consist mainly of high frequency everyday or job-related language. I can understand the description of events, feelings and wishes in personal letters.	I can read articles and reports concerned with contemporary problems in which the writers adopt particular attitudes or viewpoints. I can understand contemporary literary prose.	I can understand long and complex factual and literary texts, appreciating distinctions of style. I can understand specialised articles and longer technical instructions, even when they do not relate to my field.	I can read with ease virtually all forms of the written language, including abstract, structurally or linguistically complex texts such as manuals, specialised articles and literary works.
SPEAKING	Spoken Interaction	I can interact in a simple way provided the other person is prepared to repeat or rephrase things at a slower rate of speech and help me formulate what I'm trying to say. I can ask and answer simple questions in areas of immediate need or on very familiar topics.	I can communicate in simple and routine tasks requiring a simple and direct exchange of information on familiar topics and activities. I can handle very short social exchanges, even though I can't usually understand enough to keep the conversation going myself.	I can deal with most situations likely to arise whilst travelling in an area where the language is spoken. I can enter unprepared into conversation on topics that are familiar, of personal interest or pertinent to everyday life (e.g. family, hobbies, work, travel and current events).	I can interact with a degree of fluency and spontaneity that makes regular interaction with native speakers quite possible. I can take an active part in discussion in familiar contexts, accounting for and sustaining my views.	I can express myself fluently and spontaneously without much obvious searching for expressions. I can use language flexibly and effectively for social and professional purposes. I can formulate ideas and opinions with precision and relate my contribution skilfully to those of other speakers.	I can take part effortlessly in any conversation or discussion and have a good familiarity with idiomatic expressions and colloquialisms. I can express myself fluently and convey finer shades of meaning precisely. If I do have a problem I can backtrack and restructure around the difficulty so smoothly that other people are hardly aware of it.
	Spoken Production	I can use simple phrases and sentences to describe where I live and people I know.	I can use a series of phrases and sentences to describe in simple terms my family and other people, living conditions, my educational background and my present or most recent job.	I can connect phrases in a simple way in order to describe experiences and events, my dreams, hopes and ambitions. I can briefly give reasons and explanations for opinions and plans. I can narrate a story or relate the plot of a book or film and describe my reactions.	I can present clear, detailed descriptions on a wide range of subjects related to my field of interest. I can explain a viewpoint on a topical issue giving the advantages and disadvantages of various options.	I can present clear, detailed descriptions of complex subjects integrating sub-themes, developing particular points and rounding off with an appropriate conclusion.	I can present a clear, smoothly-flowing description or argument in a style appropriate to the context and with an effective logical structure which helps the recipient to notice and remember significant points.
WRITING	Writing	I can write a short, simple postcard, for example sending holiday greetings. I can fill in forms with personal details, for example entering my name, nationality and address on a hotel registration form.	I can write short, simple notes and messages. I can write a very simple personal letter, for example thanking someone for something.	I can write simple connected text on topics which are familiar or of personal interest. I can write personal letters describing experiences and impressions.	I can write clear, detailed text on a wide range of subjects related to my interests. I can write an essay or report, passing on information or giving reasons in support of or against a particular point of view. I can write letters highlighting the personal significance of events and experiences.	I can express myself in clear, well-structured text, expressing points of view at some length. I can write about complex subjects in a letter, an essay or a report, underlining what I consider to be the salient issues. I can select a style appropriate to the reader in mind.	I can write clear, smoothly-flowing text in an appropriate style. I can write complex letters, reports or articles which present a case with an effective logical structure which helps the recipient to notice and remember significant points. I can write summaries and reviews of professional or literary works.

Immersion or integration? What are we doing?

(adapted from Jaana Seikkula-Leino: *CLIL Learning: Achievement levels and Affective factors:* 2007)

The differences between integration and immersion are:

1 In CLIL pupils learn to read and write in L1 (unless immigrants).
In immersion (as in Canada/USA) pupils learn to write in L2 (or L2 plays a large role in the process)

2 Immersion teachers often have a different L1 from their pupils, in CLIL, the teachers share the same L1 as their pupils

3 In CLIL there should be no requirements of the pupils L2 competence in advance of the course

4 In immersion at least 50% of the teaching is in L2.
In CLIL, according to EU, this figure is 25%.

5 Immersion aims for the pupils to become fluent in both languages: in CLIL, this aim is not so ambitious.

6a The driving force behind many immersion programmes is improvement of language proficiency.
in CLIL, the driving force should be increased motivation, participation, greater inclusivity and accessibility to both content and language

6b Research into the success of the Immersion programmes often focuses on language proficiency only.
In CLIL, success should be judged by progress and proficiency in both content and language

7 Many immersion courses are selective, CLIL courses should be inclusive.

8 Many immersion courses teach fairly traditionally (Dieter Wolff).
CLIL should use bottom up, co-operative and cognitive activities.

9 In many immersion programmes the L1 of the pupil is not encouraged/developed/used in the lesson: in CLIL this development is actively encouraged.

Tenses needed for content understanding and reproduction

CLIL subject teachers often worry that they don't know enough about English grammar and tenses. Don't worry!
Tenses are like tools in a tool-box. You only use the one you need for the job. Some tools you never use because you never do those jobs! Just learn how to use the ones for your subject.

Science (including some Geography)	**History**
1 Present simple 2 Present simple passive 3 Past simple passive (experiments) 4 'will' future'	1 past simple 2 past perfect 3 past passive 4 third conditional

Check your textbooks now!

1 Get into the habit of noticing which tense is dominant in texts you ask students to read (check with ELT teacher that students have covered it)

2 Before reading a text ask students to look at the tense used before they process the content

3 Ask students to think about the tenses they need before starting a piece of writing, for example: Look at the questions below. What tense/s will you need to write the answers?

Geography 1 What is the difference between weather and climate? 2 Will a rain gauge in Oxford give a useful measurement when studying rainfall in UK? Explain your answer 3 What effect do you think straightening a river might have? 4 Why are oil-fired power stations and nuclear power stations found on the coast?	**Biology** 1 Explain why blood in arteries is a brighter red than the blood in veins 2 Why is cross-pollination usually better than self- pollination? 3 What special problems would mammals have in using evaporation to keep cool if they lived in a dry desert? 4 The River Thames used to contain salmon but until recently there had been no salmon for many years. Why was this?
Chemistry 1 Why is carbon dioxide a very important gas for living organisms? 2 Describe why substances are more likely to react together when they are in a solution than in solid form. 3 What would happen if you cooled a saturated solution of copper sulphate from 70 degrees C to 10 degrees C?	**History** 1 Which facts about Mussolini's appearance can you find in the source? 2 What did Stalin think would happen if the allies invaded France? 3 Describe what happened in eastern Europe after 1945 4 Explain why the first world war was different from all previous wars

conditional

Grammar needed to handle CLIL texts

Grammar structure	Examples
1 present passive+ prepositions and conjunctions	*The sticky plastic liquid is forced through spinnerets..* *Synthetic fibres are made from chemicals .*
2 past passive + prepositions and conjunctions	*The hydrogen supply was turned on and was allowed to pass through the test tube before...* *Some plastics can be made into monofilaments*
3 gerund as noun	*Using a plumb line shows the direction of the Earth's gravitational pull*
4 use of 'depends on whether' If	*The direction of the force depends on whether the charges are like or unlike.* *Successful conception depends on whether..*
5 Familiarity with 'although'	*Although* *iron had been made. .. iron manufacturers had difficulties* *Although they had promised to support the invasion..*
8 Familiarity with 'time' phrases	*At this time, at the time, at that time,* *on time, in time, time after time...*
9 Ease with anaphoric reference	*A plant needs light to make food. So it responds to it by growing towards it*
10 All conditionals	*The temperature of the gas should be well above that of the critical temperature* *What could happen if the gas pressure were above the critical pressure?* *What would have happened if the kinetic energy of the molecules has been converted?*
11 Understanding difference between fact and opinion	*The temperature of the liquid should have been higher* *The experiment could have been carried out in a more efficient way.* *The experiment would have been more successful if..* .
12 Past perfect simple	*For some time Lenin had wanted to...* *The Austrians had supplied the Bolsheviks with money*
13 Past perfect progressive (as factual/ emotional comment)	*The war had been raging for three years when...* *The peasants had been leaving the countryside in their thousands because of...*

Bisogna far capire ecchi o cose si riferiscono i termini that, Those, it, There ... in un testo (em CLIL)
Es: The cat sat on the mat. It was on the floor. It was very old. Mrs Smith licked it a lot. Le chi si riferiscono el gatto o al tappeto?

Collegiality between subject and English language teachers

Please add in any other factors or queries relevant to your own subject, department, year or school situation

"All teachers are language teachers"

Relationship: key questions 1 Do I get on with this person? 2 Shall we work formally or informally? 3 How shall we mark our specialist territory? 4 How can we equalise the relationship? 5 How we prevent transferring blame?	**Subject teachers responsibilities**: 1 subject specific vocabulary 2 text types/genres common to their subject 3 Language functions required by their subject
English Language teachers responsibilities 1 Listening skills 2 Reading skills 3 Note taking skills 4 Study Skills 5 Spelling and punctuation 6 Grammar [7 Pronunciation] 8 Vocabulary expansion – academic [CALP] rather than social [BICS] 9 pair/group work skills 10 self-evaluation 11 development of learner strategies	**Organisation** 1 Decision making forum – where? When? how? What? 2 Possible syllabus changes particularly for language teachers 3 Possible timetable changes 4 How to reduce preparation time? 5 How to share resources? 6 How to obtain resources? 7 assessment?

Benefits of collegiality for teachers and administrators

Diana Hicks

(adapted from Shah,M 2012, Elsevier Ltd ***The importance and benefits of teacher collegiality in schools – a literature review)***

Benefit	**I thought of**	**Surprised me**
1 improves teacher professional growth		
2 increases teacher professionalism		
3 raises school quality		
4 improves organisational effectiveness		
5 improves student behaviour, attitude and achievement		
6 encourages innovation		
7 uses a greater range of resources and methods		
8 reduces emotional stress and burnout		
9 encourages a sense of 'belonging'		
10 increases motivation		
11 increases career commitment		
12 lowers staff turnover		
13 makes changes/aspirations more realistic		
14 stimulates energy		
15 increases capacity for professional reflection		
16 reduces 'overloadedness'		
17 increases trust		
18 raises collective responsibility for student achievement and well being		
Others		

Different kinds of Cognition- Bloom's Taxonomy

Taxonomy of Educational Objectives: Bloom, B. S : 1984 Pearson

1 When planning your lessons or units of work, ask yourself 'How often am I asking my students to do these kinds of cognitive tasks?'

Action research: Make a copy of this sheet for each class and tick off each type of thinking skill for each task.

1 Assessing	28 Illustrating
2 Calculating	29 Integrating
3 Changing	30 Interpreting
4 Classifying	31 Inventing
5 Collecting	32 Judging
6 Combining	33 Labelling ×
7 Comparing	34 Linking
8 Completing	35 Listing
9 Composing	36 Measuring
10 Concluding	37 Modifying
11 Contrasting	38 Planning
12 Creating ×	39 Predicting
13 Deciding	40 Preparing
14 Defining	41 Quoting
15 Describing	42 Ranking
16 Designing	43 Re-arranging
17 Differentiating	44 Recommending
18 Discovering	45 Relating
19 Discussing	46 Rewriting
20 Estimating	47 Selecting
21 Examining	48 Showing
22 Experimenting	49 Solving
23 Explaining	50 Substituting
24 Formulating	51 Summarising
25 Generalising	52 Telling
26 Grading	53 Testing
27 Identifying	54 What if?

2 Write some activities for your subject for each kind of cognition

CLIL Lesson/unit of Work structure

Stage	Strategies	Skills used- Bloom's taxonomy
1 Tuning in	Considering possibilities : What would happen if...?' Children asking questions Pair/group discussion Making drawings/diagrams	Questioning Organising Sharing ideas Listening Talking Planning Predicting Estimating Querying
2 Finding out	Visitors Outside visits Videos Research on web/CD roms Reading Experiments	Observing Summarising Asking questions Selecting information researching Reading Note taking Finding resources Collaborating Comparing and contrasting Making connections Using IT Making decisions and choices Time management
3 Sorting out	Classifying/grouping/sorting Charts/graphs/ sequencing – narrative/process	Organising Classifying Seeing links/patterns Collaborating Presenting ideas Talking Listening Reading Interpreting Writing Drawing Reporting Testing Inventing Designing Using IT Making choices
4 Reflecting	Self evaluation and assessment Peer evaluation and assessment Learning journals Public display/publicity Teacher evaluation/assessment	Responding Speaking and writing clearly Performing Clarifying

Reflection questions

Diana Hicks

The last part of every lesson should be given over for student reflection.

Reflection should be done in Mother Tongue – this is not a language exercise but a means of improving the ways of learning for each student.

At the end of each lesson choose two of the following questions and put them on the board- some will be more appropriate to your students than others.

Students choose one and reflect. They can also this as a basis for their 'exit pass'.

Reflection questions

1 What skills have you practised/developed today?
2 What questions did you ask today to help you achieve your best?
3 What might a friend or a teacher say about your work now?
4 How do you know you are making progress?
5 Where do you need more help/practice?
6 What activity today helped you understand/learn best? Why?
7 What can you teach someone else from today's lesson/topic?
8 What stopped you from learning/doing your best today?
9 Who stopped you from focussing/improving today?
10 What can you take away from this lesson?
11 Have you learned anything more about your ways of learning today?
12 Which activity today helped you the least?
13 Which activity did you find most difficult today? Why?
14 Is there anything from the lesson you would like to share with anyone else?
15 Did the lesson/activities build on anything you knew before?
16 Did anything surprise you in your learning or the topic today?
17 What changes to the lesson would help you better next time?
18 What strategies can you use to help you understand the lesson better?
19 What can you teach someone else now?
20 What do you know now that you didn't at the beginning of the lesson?

What is a cognitive task?
From Bentley, K The TKT Course (CUP 2009)

Work alone or with a partner. Look at the tasks in each box. Tick the ones which you think require cognition.

1Comparing	A Write down three facts about how fossil fuels are used. B Tell your partner three differences between coal and oil C Look at the table about coal and oil and list the similarities in how they are produced
2 Predicting	A What could happen if we add more H2SO4? B Scan the text and underline the stages of development C Look at the pictures of the objects found in the tomb. What can you say about the person's life?
3 Reasoning	A Label the electrical components and copy the circuit diagram B Put the words in the box into a Venn diagram C Study the life cycle of the flowering plant and explain to a partner how the seeds germinate
4 Classifying	A Put the pictures into groups. What features do they have in common? B decide how these sources could be grouped and explain why C Read the text and make a mind map showing the key features of the terrain
5 Sequencing	A Place these events on the timeline B Some of these events are in the wrong chronological order. Correct them. C Put these events leading up to the First world war into order of importance (not chronology)
6 Evaluating	A Suggest an improvement to the experiment design B Compare the charts your group has produced. Which one shows the information most clearly? Why and how? C Which part of your work would you like to improve? How could you do this?

Topical Applications of Mathematics

Genetic Fingerprinting — PUPIL TEXT

Genetic (or **DNA**) **fingerprinting** was developed by Professor Sir Alec Jeffreys at the University of Leicester in 1984. The technique is based on the fact that each of us has a unique sequence or code of genetic information, contained in our DNA (deoxyribonucleic acid) in the nucleus of every living human cell. This is inherited from our natural parents, half from our mother and half from our father.

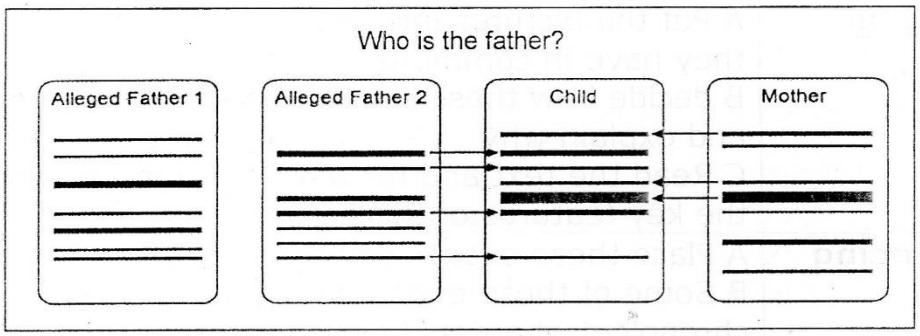
Although the majority of DNA doesn't differ from human to human, about 0.10 percent of a person's entire genome (the total set of genes carried by an individual or cell) varies from person to person. These areas of DNA, called **minisatellites** (short sequences of chemical building blocks) show variation in the numbers of repeat units (or stutters) unique to each person. These form the bands that are illustrated below.

DNA information can be recovered from human and animal remains as far back is Neanderthal man and has been used to solve a number of high profile mysteries from the past.

Apart from identification, paternity and immigration cases, the technique is also used in medical research including cancer and genetic conditions such as Huntington's disease.

Unless you have an identical twin, your DNA is unique to you. This is what makes DNA evidence so valuable in investigations - it's almost impossible for someone else to have DNA that is identical to yours.

The sketch below shows, in simplified form, how genetic fingerprinting can be used to identify a child's father.



It is usual to compare between 10 and 20 bands. Experimental evidence has shown that in unrelated people, the probability of one band matching is one in four, a probability of 0.25.

So, for example, the probability of two bands out of two matching

$$= 0.25^{2}$$

$$= 0.0625 \text{ or a 1 in 16 chance.}$$

Activity 1

Find the probability of 10 bands out of 10 matching. Express your answer in the form,

"1 in ? chance".

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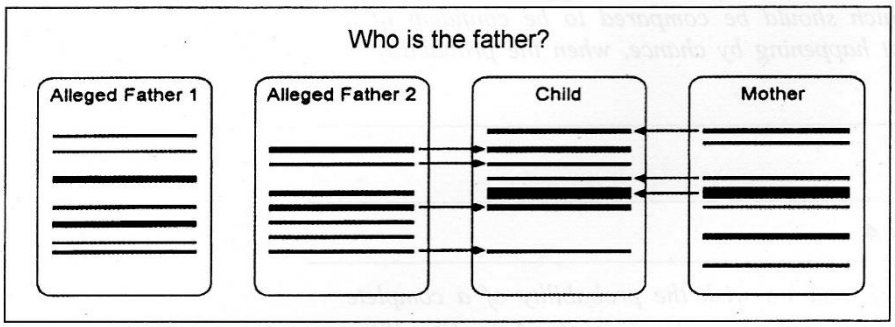
Unless you have an identical twin, your DNA is unique to you. This is what makes DNA evidence so valuable in investigations - it's almost impossible for someone else to have DNA that is identical to yours.

The sketch below shows, in simplified form, how genetic fingerprinting can be used to identify a child's father.



It is usual to compare between 10 and 20 bands. Experimental evidence has shown that in unrelated people, the probability of one band matching is one in four, a probability of 0.25.

So, for example, the probability of two bands out of two matching

$$= 0.25^{2}$$

$$= 0.0625 \text{ or a 1 in 16 chance.}$$

Activity 1

Find the probability of 10 bands out of 10 matching. Express your answer in the form,

"1 in ? chance".

Activity 2

Repeat Activity 1 but using 0.5 as the probability of any single band matching".

You will have noticed that the answers to Activities 1 and 2 change quite dramatically if the underlying probability changes. In fact, the value of 0.25 was the subject of some speculation in a number of criminal trials but it has been established now as an acceptable value to use.

Activity 3

Copy and complete the table below.

Probability	*Number of bands compared*			
(p)	5	10	15	20
0.2	1 in 3125	?	?	1 in 95 million million
0.25	?	?	?	?
0.5	?	?	?	?

Comment on the values found and suggest the number of bands which should be compared to be confident of a match not happening by chance, when the probability is 0.25.

Activity 4

If $p = 0.25$ *and we wish the probability of a complete match* ***not*** *happening by chance to be 1 in 60 million (approximately the population of Britain), how many bands need to be compared?*

Topical Applications of Mathematics

Genetic Fingerprinting TEACHER INFORMATION

Key Stage 3 or 4

Target High ability KS3 pupils and Higher level GCSE students

MEP references GCSE Unit 5

Teaching notes This resource is highly relevant to real-life situations and is, at its heart, based on straightforward probability. It could be used with KS3 pupils but the range of the applications (for example, not just paternity but many high profile murder and rape cases; it played a major part in the prosecution and conviction in 2008 of the murderer of five prostitutes in Ipswich) is wide and instances where genetic fingerprinting is used are extensive and of interest to all. The material presented here attempts only to explain the part played by probability. You might consider working with your Biology teachers in order to cover in more detail the background to DNA.

Useful references include those at

http://www.parliament.uk/documents/upload/postpn258.pdf

and

http://www.forensic.gov.uk/forensic_t/inside/news/fact_sheets.htm

An interesting lecture by the originator of the method, Professor Sir Alec Jeffreys, can be found at

http://www.ntu.ac.uk/news/events/57767gp.html

Another avenue to explore would be to invite the forensic officer from your local police force to talk to your students about this application (and indeed other techniques used to solve crimes).

Solutions and Notes for material in the Pupil Text

Activity 1

There is about a 1 in 1 million chance of 10 out of 10 bands matching.

Activity 2

There is about a 1 in 1000 chance of 10 out of 10 bands matching.

Activity 3

p	5	10	15	20
0.2	1 in 3125	1 in 9.8 million	1 in 30 thousand million	1 in 95 million million
0.25	1 in 1024	1 in 1 million	1 in thousand million	1 in 1.1 million million
0.5	1 in 32	1 in 1024	1 in 32 768	1 in 1 million

Activity 4

We want $\left(\frac{1}{4}\right)^n = \frac{1}{60000000}$

You can use trial and improvement or, if familiar with logarithms, then

$$\ln\left(\frac{1}{4}\right)^n = \ln\left(\frac{1}{6\times 10^7}\right)$$

$$\ln 4^{-n} = \ln 6^{-1}\times 10^{-7}$$

$$-n\ln 4 = -\ln 6 - 7\ln 10$$

$$-2n\ln 2 = -\ln 6 - 7\ln 10$$

$$n \approx 12.92$$

So we take $n = 13$.

Topical Applications of Mathematics

Genetic Fingerprinting — SAMPLE LESSON PLAN

Activity		*Notes*
		T: Teacher P: Pupil
1	**Introduction** T: Who knows anything about DNA? *(Ps volunteers their knowledge and understanding).* T: What is the mathematical basis for Genetic Fingerprinting? Look at *Data Sheet 1.* T: The 'bands' of your unique DNA are made up of 50% from your mother and 50% from your father. Is your DNA unique? *(Yes, unless you have an identical twin!)* T: It is estimated that the chance of having one band in common is $\frac{1}{4}$. This is though an experimental value and has been the subject of recent debate in court cases. T: So what is the probability of matching 2 out of 2 bands compared? P: $\left(\frac{1}{4}\right)^2 = \frac{1}{16} = 0.0625$ T: Can you express this as a '1 in ?' chance? P: 1 in 16 T: That's right. This probability is not sufficient to avoid error so we need to look at matching more bands. *10 mins*	It is best if this topic has been introduced recently in Biology lessons. If not, you need to capture the key points, moving on to the mathematical underpinning rather than some of the moral aspects. Put Data Sheet 1 on OHP to clarify the concept of matching bands.
2 *(continued)*	**Matching more bands** T: Work in pairs for 2 minutes to find the probability of matching (a) 10 out of 10 bands (b) 20 out of 20 bands T: Who will show us their answer on the board? P (at board): $\left(\frac{1}{4}\right)^{10} = \frac{1}{1048576}$ About 1 in 1 million chance	Make sure that Ps understand the problem; intervene if necessary. Choose Ps to show solution on the board.

Activity		*Notes*
2 *(continued)*	P (at board): $= \ln 6^{-1} \times 10^{-7} \left(\frac{1}{4}\right)^{20} \approx \frac{1}{1.09 \times 10^{12}}$ About 1 in 1.1 million million chance. *20 mins*	Some discussion may be needed on accuracy and how best to present the answers.
3	**Different probabilities** T: The value of $\frac{1}{4}$ for matching one band at random is only experimental. Investigate the effect that varying this value has on the results. T: How can we cope with this? T (after about 5 minutes): What can you conclude? P: The technique is very sensitive to the value assumed for p. *30 mins*	You can make this as open as you like, but if you want to tie it down, then use Data Sheet 2, in which $p = 0.2$ and 0.5 is compared with $p = 0.25$ for 5, 10, 15 and 20 bands. Give Ps time to analyse the results and ensure that they all take part in the discussions.
4 *(continued)*	**Extensions** T: Assume that it is safe to take $p = \frac{1}{4}$. The population of the UK is about 60 million. What is the number of bands that need to be compared to ensure that it is safe to convict on DNA evidence alone? You have 10 minutes to find your answer. T (after about 10 minutes): Who has a method? What do we need to do? P: We need to solve $\left(\frac{1}{4}\right)^n = \frac{1}{60000000}$ to obtain the value of n. T: How can we solve this? P: Trial and improvement. T: Yes, but if we use logarithms we can get the answer quickly. We know that $\ln\left(\frac{1}{4}\right)^n = \ln\left(\frac{1}{6 \times 10^7}\right)$ $\ln 4^{-n} = \ln 6^{-1} \times 10^{-7}$ $-n \ln 4 = -\ln 6 - 7 \ln 10$	You might want to give more help or discuss the approach here. Monitor progress, intervening if necessary; working in pairs should be encouraged. Help Ps to find a method of solution. Use this method if they have covered logs; otherwise intelligent use of trial and improvement is needed.

Activity **4** *(continued)*	$-2n\ln 2 = -\ln 6 - 7\ln 10$ $n = \frac{1}{2\ln 2}\ln 6 + 7\ln 10$ ≈ 12.92 So we take $n = 13$ *45 mins*	***Notes*** You need to stress that rounding up is needed to ensure the result is valid.

3.3 Tell me what you know

Aims LANGUAGE Asking and answering questions (see Appendix 2).
OTHER Using questionnaires; oral revision; interviewing skills.

Demo subject HISTORY
TOPIC Revision of studied period, for example, the 1960s

Alternative subjects
PHYSICS Waves
GEOGRAPHY Climate
BIOLOGY Photosynthesis and the nutrition of green plants
Any subject area

Preparation

Write a worksheet and make copies, one for each student (see the example on the next page). Write an answer key.

Procedure

1 Give each student a copy of the worksheet. Set a time limit. Ask the students to walk round the class and interview as many other students as possible. Tell them to try to find answers for at least eight of the questions before you stop them.
2 On the dotted lines the students make notes of the answers they get and the names of the students who gave them.
3 When they have finished choose a student to read out the information they collected for question 1. Choose a different student to answer for question 2. Continue through the worksheet. Tell the students to keep the questionnaire for future reference.

Follow-up 1

In a later lesson read the questions again and get the students to write down answers individually.

Follow-up 2

Students could work in groups to make their own 'Find someone who...' sheets to be used in the class.

Follow-up 3

Students work in groups and choose one of the questions to turn into a five-minute presentation.

Worksheet 3.3

Find someone who...

1 can name the year President J. F. Kennedy was killed.

_____

2 can give you the names of two US Civil Rights leaders in the 1960s

_____

3 knows which European country the Soviet Union invaded in the late 1960s

_____

4 knows what medical 'first', Christian N. Barnard achieved in 1967

_____

5 can tell you which countries fought in the Six Day War

_____

6 knows when the Berlin Wall was built and demolished

_____

7 knows the names of the first moon walkers and the name of their spacecraft

_____

8 can name a famous sportsperson of the 1960s

_____

9 can name some famous musicians, artists and writers of the 1960s

_____

10 can name a fashion designer and describe the kinds of clothes worn in the 1960s

_____

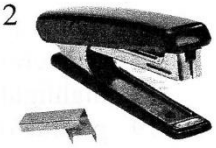
Answers to questions 1–8

1 1963
2 Martin Luther King and Reverend Abernathy
3 Czechoslovakia
4 heart transplant
5 Israel and Palestine
6 1961 and 1989
7 Neil Armstrong and Buzz Aldrin in Apollo 11

Questions 8, 9, and 10 have numerous answers

Classroom objects

1 

2 

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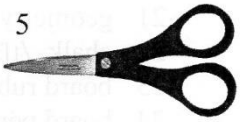
5 

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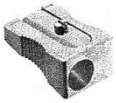
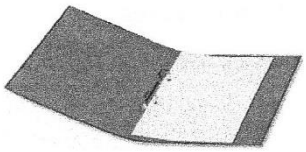
9 

10 

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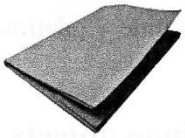
12 

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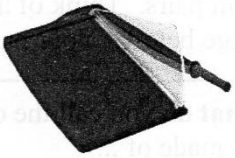
15 

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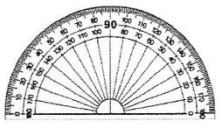
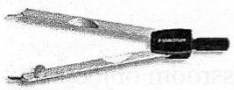
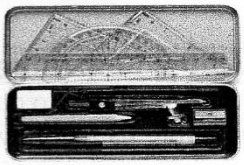
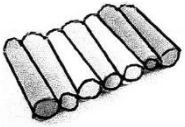
19 

20 

21 

22 

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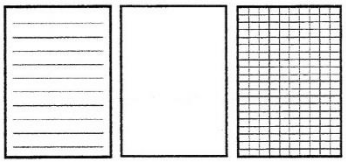
25 

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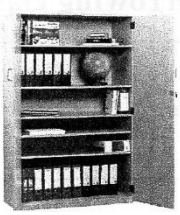
29 

30 

Classroom objects

1 hole punch(er)
2 stapler and staples /ˈsteɪplə(r)/
3 pocket calculator
4 ruler (plastic, wooden, metal)
5 (a pair of) scissors /ˈsɪzəz/
6 Sellotape
7 paper clip
8 pushpin / drawing pin
9 pencil sharpener
10 ring file / ring binder
11 plastic files
12 paper file
13 teacher's desk with drawers
14 data projector
15 guillotine /ˈgɪləti:n/
16 rubber (UK) /eraser (US)
17 correction fluid
18 highlighter pen
19 protractor /prəˈtræktə(r)/
20 a compass / a pair of compasses
21 geometry set
22 chalk /tʃɔ:k/
23 board rubber (UK) board eraser (US)
24 board pen
25 lined, plain and squared paper
26 scrap paper
27 glue stick
28 overhead projector
29 classroom cupboard with shelves
30 screen

Describing objects

Work in pairs. Think of a classroom object and describe it to your partner using the language below. Your partner must guess what it is.

- **What** do you call the object for … + *-ing*?
- It's made of ...
- It's for ... + *-ing*
- You use it for ... + *-ing*
- **What** do you call it?

+ It's a ...

Describing how things work

Work in pairs. Point to one of the classroom objects and ask your partner what it is for. Then ask your partner how it works.

+ What's this for?

- It's for ... + *-ing*
- It's used for ... + *-ing*
- It's a device/instrument for ... + *-ing*

+ Oh, I see. **How** does it work?

Useful classroom language – borrowing classroom objects

+ Can I borrow your … , please?
- Yes of course you can. Here you are.
+ Thank you.

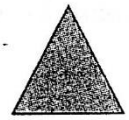
Maths shapes

Two-dimensional shapes

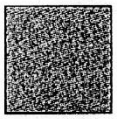
circle
/ˈsɜːkl/

semicircle
/ˈsemi sɜːkl/

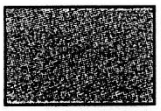
ellipse/oval
/ɪˈlɪps/ /ˈəʊvl/



triangle
/ˈtraɪæŋgl/

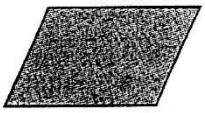


square
/skweə(r)/



oblong/rectangle
/ˈɒblɒŋ/ /ˈrektæŋgl/

rhombus
/ˈrɒmbəs/



rhomboid
/ˈrɒmbɔɪd/

trapezium
/trəˈpiːziəm/

trapezoid
/ˈtræpəzɔɪd/

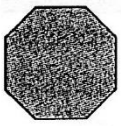
cross
/krɒs/

star
/stɑː(r)/

pentagon
/ˈpentəgən/

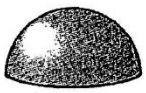
hexagon
/ˈheksəgən/

heptagon
/ˈheptəgən/



octagon
/ˈɒktəgən/

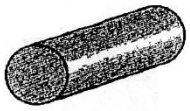
Three-dimensional shapes



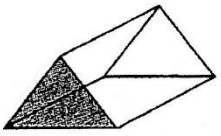
hemisphere
/ˈhemɪsfɪə(r)/

sphere
/ˈsfɪə(r)/

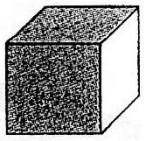
cone
/kəʊn/



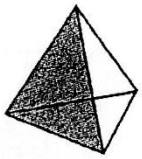
cylinder
/ˈsɪlɪndə(r)/



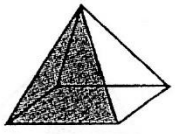
prism
/ˈprɪzəm/



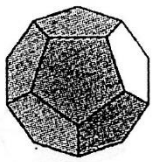
cube
/kjuːb/



tetrahedron
/tetrəˈhiːdrən/



pyramid
/ˈpɪrəmɪd/



dodecahedron
/dəʊdekəˈhiːdrən/

47

Saying numbers (1)

Mathematical signs

What do these signs mean and how do you say them?

\+ plus (plas) add and

− minus (meinus) subtract Take away less

x times a multiplied by

÷ divided by

= equals is equal to is makes como (por ejemplo el cuadrado el segundo)

+ = addition − = subtraction x = multiplication ÷ = divide

Speaking practice: I like doing sums!

Work with your partner. Take turns telling one another what mathematical signs are missing below.

3 3 = 9	12 3 = 36
6 7 = 13	13 25 = 38
24 4 = 6	9 9 = 81
8 7 = 56	120 60 = 60
18 2 = 9	21 8 = 13
100 10 = 10	33 11 = 3
17 5 = 12	15 4 = 60
4 5 = 20	12 3 8 = 23
16 4 = 4	10 5 3 = 12
4 2 = 2	3 3 2 = 18

Now make some similar sums to test your partner.

Times tables

You say the threes times table like this: "One three is three, two threes are six, three threes are nine," etc. Practise saying the 4 times table with your partner. **PTO**

Types of number

2, 4, 6, 8, 10, etc. are even numbers.

1, 3, 5, 7, 9, etc. are odd numbers or uneven numbers.

Number sequences

What are the next two numbers in the sequences? Explain why to your partner.

3, 7, 11, 15, 19, 23 27 ARITHMETIC PROGRESSION

1, 2, 4, 8, 16, 32 64 GEOMETRIC PROGRESSION

1, 2, 4, 7, 11, 16 22

1, 1, 2, 3, 5, 8, 13, 21 34 FIBONACCI SEQUENCE

2, 6, 18, 54, 162, 486 1458 GEOMETRIC PROGRESSION

1, 4, 9, 16, 25, 36 49 SQUARED NUMBERS

Do these sequences have names?

Number boxes

Use each number in the box only ONCE to produce a result of 24.

9	4
10	7

= 24

Answer: 10-9 = 1, 7-1 = 6, 6x4 = 24, **or** 9-7 = 2, 2 x 10 = 20, 20 + 4 = 24, **or** 4x10 = 40, 40-9 = 31, 31 - 7 = 24

4	5
10	7

= 24

4	8
9	10

= 24

1	10
4	3

= 24

9	5
2	6

= 24

Using quadratic equations to solve problems

Student A **Example 1**

1. The product of two consecutive even numbers is 120. What are the numbers?

2. Let the first number be x the second even number is Two more than x and can be written as $x+2$

3. Then $x \times (x + 2) = 120$

4. Multiple out the brackets : $x^2+2x=120$

5. Rearrange to equal zero: $x^2 + 2x - 120 = 0$

6. Factorise : $(x-10)(x+12)=0$

7. $x = 10$ or $x = -12$

8. So, the two number are 10 or 12 or -12 and -10

9. There are two possible answers.

10. Answering the question by "Trial and improvement" would find the positive answer, but probably not the negative answer.

When you have finished:

a) Compare and check what you have written with your partner.

b) What does the word "Then" mean in line 3?

c) What does the word "So" mean in line 8?

5

Using quadratic equations to solve problems

Student B **Example 1**

1. The product of Two consecutive even numbers is 1020. What are the numbers?

2. Let the first number be x. The second even number is two more than x and can be written as $x + 2$.

3. Then $x \cdot (x+2) = 120$

4. Multiply out the brackets: $x^2 + 2x = 120$

5. Rearrange To equal zero : $x^2+2x-120=0$

6. Factorise: $(x - 10)(x + 12) = 0$

7. So $x = 10$ or $x = -12$!

8. So, the two numbers are 10 and 12 or −12 and −10.

9. There are Two possible answers

10. Answering the question by "trial and improvement" would find the positive answer, but probably not the negative answer.

When you have finished:

a) Compare and check what you have written with your partner.

b) What does the word "Then" mean in line 3?

c) What does the word "So" mean in line 8?

Saying numbers (2)

Exercise 1
How do you say the following numbers?

1 $11 + 3 - 6 = 8$; $7 \times 6 = 42$; $27 \div 3 = 9$

2 0.0305; 0.2222222222 (correct to 10 dp) or $0.\overline{2}$
nought point

3 ½; ¼; ¾; ⅛; ⅝; ⅜

4 $\frac{1}{7} \approx 0.142857$

5 $\frac{x}{y}$ (x over y)

6 2,718; 1,618

7 $\pi \approx 3.14$; $e \approx 2.718$; $\varphi \approx 1.618$
↑ is approximately equals to

8 43.5%

9 -35°C; 89°F; $0°C \equiv 32°F$
↑ identical to

10 $2\pi r$; πr^2
2 pi r

11 x^2; x^3; x^n

(10 The power of n

x squared x cubed

12 $\frac{1}{1000} = 0.001 = 10^{-3}$

13 $\sqrt{8} \approx 2.828$

The squared root of 8

14 $\sqrt[3]{8} = 2$

The cube root of 8

15 $\sqrt[n]{x}$

the nth-root of x

16 4:3

The ratio of four to 3

17 $x \neq 0$; $\therefore x \to \infty$; $\because y = 3$

→ because

↓ i.e.

Therefore finish.

18 $x > y$; $y < z$; $3 < x < 4$

x is greater than y less than z x between

19 $x + y + z \leq 1$; $n \geq 1$

20 $x \propto y$; Intensity $\propto \frac{1}{\text{distance}^2}$

Saying numbers (2) (pages 71-73)

1 eleven plus (add) three minus (take away, subtract, negative) six equals (is equal to, makes, is) eight; seven times (multiplied by) six equals forty two; twenty seven divided by three equals nine.
+, – , x and ÷ are *operators* and are symbols to show which operation needs to be done.

2 nought /nɔ:t/ point oh three oh five; nought point two two two etc. correct to ten decimal places, or nought point 2 repeating/recurring. This is a repeating/recurring/periodic decimal. The bar can be above a single repeating digit or repeating block of digits.

3 a half, a quarter, three quarters, one eighth, five eighths, nine eighths (It is also possible to say one over two, one divided by two, etc.). These are *common fractions*. The line that separates the fraction is called the *fraction bar*. The top number is the *numerator* and the bottom number is the *denominator*. If the numerator is smaller than the denominator it is a *proper fraction*. If the numerator is larger than the denominator, it is an *improper fraction*.

4 one over seven (or the *reciprocal* of seven) is approximately equal to nought point one four two eight five seven.

5 *x* over *y*, or *x* divided by *y*, or *x* upon *y* (less common).

6 two thousand seven hundred and eighteen; one thousand six hundred and eighteen. Note that the word "hundred" is followed by "and". Note also that the comma signifies thousands. However, in the SI (Système International) system of units, the comma denotes the decimal marker.

7 pi /paɪ/ is approximately equal to three point one four; *e* is approximately equal to two point seven one eight (and is known as Euler's number/constant, or Napier's constant, and is the base of natural logarithms); phi /faɪ/ is approximately equal to one point six one eight (and is the *golden ratio/mean/section*). Note that we say a decimal is "correct to three decimal places (3dp)".

8 forty three point five per cent

9 minus thirty five degrees Celsius/centigrade; eighty nine degrees Fahrenheit; zero degrees Celsius/centigrade is identical to thirty two degrees Fahrenheit.

10 two pi /paɪ/ r (the circumference of a circle); pi r squared (the area of a circle)

11 *x* squared (or *x* multiplied by itself, or *x* to the power of two, or *x* to the second power); *x* cubed (or *x* to the power of three, etc.); *x* (raised) to the power of *n*, *x* to the *n*th power, *x* to the *n*th. The word *power* is identical to *exponent*.

12 one thousandth equals nought point oh oh one equals ten to the (power of) minus (negative) three. NB: 1/1000 is a *decimal fraction*, i.e. the denominator is a power of ten.

13 the square root of eight is approximately equal to two point eight two eight

14 the cube root of 8 (the third root of eight) equals two

15 the nth root of *x*

16 the ratio of four to three

17 *x* is not equal to (does not equal) zero; therefore *x* tends to infinity; because *y* equals three [Note: $\neq$ $>$ $\geq$ $<$ $\leq$ are called *inequality symbols*]

18 *x* is greater than y; *y* is less than *z*; *x* is greater than 3 and less than 4, i.e. it is somewhere between 3 and 4.

19 *x* plus *y* plus *z* is less than or equal to one; *n* is greater than or equal to one

20 *x* is proportional to *y*; intensity is inversely proportional to the square of the distance from the source of that physical quantity. This is the inverse square law.

Saying numbers (3)

1 OH, ZERO, NOUGHT

The above are all ways of saying 0 in English.

We say oh	after a decimal point	5.03	five point oh three
We say nought	before the decimal point	0.02	nought point oh two
We say zero	for the number for temperature	0 -5°C	the number zero five degrees below zero

Now say the following:

1 The exact figure is 0.002.

4 Do we have to hold the conference in Iceland? It's 30 degrees below 0!

2 THE DECIMAL POINT

In English, we use a point (.) and not a comma (,) for decimals. We use comas in figures only when writing thousands.

10,001	is ten thousand and one.
10.001	is ten point oh oh one.

In English all the numbers after a decimal point are read separately:

10.66	ten point six six
0.325	nought point three two five
0.001	nought point oh oh one

You will also hear people say:

0.05	zero point oh five OR oh point oh five

But if the number after the decimal point is a unit of money, it is read like a normal number:

£12.50	twelve pounds fifty

Now say the following:

1 It's somewhere between 3.488 and 3.491.

2 Look, it's less than 0.0001 ! It's not worth worrying about.

3 Did you say 0.225 or 0.229?

4 No, I mean 15.005 not 15,005

How to Speak Mathematics

Greek Alphabet

Capital letter	Small letter	Pronunciation									
A	α	/ælfə/	H	η	/i:tə/	N	ν	/nju:/	T	τ	/taʊ/
B	β	/bi:tə/	Θ	θ ϑ	/θi:tə/	Ξ	ξ	/saɪ/	Υ	υ	/ɪpsɪlən /
Γ	γ	/gæmə/	I	ι	/aɪəʊtə/	O	o	/əʊmaɪkrən/	Φ	φ	/faɪ/
Δ	δ	/deltə/	K	κ	/kæpə/	Π	π	/paɪ/	X	χ	/kaɪ/
E	ε	/epsɪlən/	Λ	λ	/læmdə/	P	ρ	/rəʊ/	Ψ	ψ	/psaɪ/
Z	ζ	/zi:tə/	M	μ	/mju:/	Σ	σ ς	/sɪgmə/	Ω	ω	/əʊmɪgə/

Substitution tables

Reading mathematics

1, 206, n, π, …	plus			1, 206, n, π, …	equals / is
	minus				
	multiplied	by / (with)			
	times				
	divided	by / (through)			
	squared / cubed				
	(raised) to	the	power of	0, 1, 206, n, π, …	
			4^{th}, …, n^{th}	power	
The square root / The cube root / The 4^{th} root / The n^{th} root	of	1, 206, n, π, …			

Giving instructions

Add / Subtract / Multiply / Divide / Square / Cube / Raise	1, 206, n, π, …	to / from / by / (with) / by / (through)		1, 206, n, π, …
		to the	power of 0, 5, a, …, z	
			1^{st}, 2^{nd}, 3^{rd} / 4^{th}, …, n^{th}	power
Take	the square root / the cube root / the 4^{th} root / the n^{th} root	of	1, n, π, …	

List of Typical Symbols that May Cause Problems to Read Aloud

Symbol	Meaning / pronunciation	Example
0	nought, zero, /əu/	
=	equals / is equal to	Five plus three equals eight / Area A is equal to area B.
$\neq$	does not equal / is not equal to	
$\approx$	is about / approximately equal to	One third is about nought (zero) point three
$\cong$	congruent to	
$\triangleq$	corresponds to	
$\sim$, $\propto$	proportional to	
<	less than / smaller than	Three is less than five
>	greater than	Five is greater than three
$\leq$	less than or equal to / smaller than or equal to	
$\geq$	greater than or equal to	
( )	(round) brackets / parentheses	
[ ]	square brackets	
{ }	curly brackets / wavy brackets	
$\parallel$	parallel to	
$\perp$	perpendicular to	
$A \Rightarrow B$	from A follows B / if A then B	
$\Leftrightarrow$	if and only if / iff / equivalent to	
$\frac{n}{d}$	fraction: n = numerator, d = denominator	$\frac{2}{3}$ = two thirds; n over d
$\sqrt{}$	square root of	The square root of nine is three
$\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[n]{}$	cube root, fourth root, nth root	The cube root of 8 is 2
$\lvert -5 \rvert$	modulus (absolute value)	The modulus of −5 is 5
$n!$	n factorial	$n! = n \cdot (n-1) \cdot (n-2) \ldots \cdot 3 \cdot 2 \cdot 1$
$\binom{n}{k}$	n choose k	$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!}$
π	/paɪ/	$2\pi r$, πr^2
$\sum_{n=1}^{5} n^2$	Sum (Greek sigma)	Sum from n equals 1 to 5 over / of n squared
$f(x)$	/ef əv eks/	
$f'(x)$	f prime of x; f dash	
$f''(x)$	f two prime of x / f double prime of x; f double dash	
$\frac{dy}{dx}$	/di: waɪ əʊvə di: eks/ (Derivative)	
$\int_0^3 f(x)\,dx$	Integral from nought (zero) to three over /ef əv eks di: eks/	

Exercises

25.1 Answer these questions.

1 What is five squared?
2 What is the next prime number after 19?
3 How is this sequence of numbers created? 3, 9, 27, 81
4 What is the aggregate of this set of test marks? 6, 8, 9, 5, 6, 7
5 If you round up 6.66, what number do you have?
6 $\frac{7}{5}$ and 4 – which is a whole number and which is a fraction?
7 In your country is tax automatically deducted from employees' earnings?
8 Is an accountant pleased or displeased if figures that he/she is checking tally?

25.2 **Dr Syal is advising one of his dissertation students who is interested in pollution in road tunnels. Complete the conversation. You are sometimes given the first letter to help you.**

Dr Syal: You could c..... the total number of private cars that use the tunnel each week, based on the day-to-day figures, and get an a..... figure for how much carbon they're all emitting.
Melissa: How p..... would that figure have to be?
Dr Syal: Oh, it doesn't have to be exact, you just need to e..... more or less what the total pollution will be. Then you can check to see if those figures t..... with the figures that have already been published for similar tunnels. And the figure won't be c..... of course; it'll go up and down depending on lots of factors such as weather conditions, average speed, etc.
Melissa: But can we say if the figures will be true for the future too?
Dr Syal: Well, we do know that the traffic growth has been c..... over the past ten years; it hasn't ever gone down, so I think you can make some useful predictions.
Melissa: Should I present each daily total as a d..... item or can I just put them all together into one figure for each week?
Dr Syal: A weekly total is fine, and you can ..... it up or ..... to the nearest 100.
Melissa: Right, OK. Thanks so much for your help.

25.3 **Rewrite these spoken sentences so that they are more appropriate for writing, using the word in italics in an appropriate form.**

1 There were fewer car accidents last year. *incidence*
2 We made a rough guess at what the final figure might be. *estimate*
3 The graph shows the results from the lowest to the highest. *magnitude*
4 A computer program helped us work out the significance of the different variables. *calculate*
5 Taking x away from y will help you arrive at the correct answer. *subtract*
6 The results from the first experiment were not the same as those we got from the repeat experiment. *tally*

25.4 **Fill in the gaps in this advice a maths lecturer is giving her students.**

In the exam, don't forget to show all your (1) ..... as we want to see how you (2) ..... at your results. Make your (3) ..... very carefully – you'd be amazed at how many people submit answers that are hardly even in the right (4) ..... . And please write legibly – we must be able to distinguish all your (5) ..... ! When doing graphs, plot your (6) ..... carefully and if asked to describe an experiment don't forget to take all significant (7) ..... into account. Good luck!

25 Numbers

A Types of numbers

Numbers in a group together may be called a **series** or **set of numbers**. If the order in which they occur is significant then they may be called a **sequence of numbers**. 1, 4, 9, 16, 25 is a sequence of numbers, for example – it represents the numbers 1 to 5 **squared**.

1, 3, 5, 7 ... = **odd numbers**; 2, 4, 6, 8 ... = **even numbers**; 2, 3, 5, 7, 11 ... = **prime numbers**. The highest number in a group is the **maximum** and the lowest is the **minimum**. *The room holds a **maximum** of 50 and we won't run the class without a **minimum** of 12 students.*

An **approximate** number is one which is roughly correct but is not the **precise** or **exact number**. *Look at the figures and work out in your head what the **approximate** answer is likely to be. Then use a calculator to find the **exact** number.*

An **aggregate** is a number reached by totalling a set of numbers = the **total**. *The average mark achieved in the exam is calculated by taking the **aggregate** of all the marks and dividing by the number of exam entries.*

A **discrete** number or unit is something which is separate and cannot be divided into smaller numbers or units of the same thing. The opposite of discrete is **continuous**. A bag of apples, for example, could be considered as consisting of discrete items whereas apple sauce could be considered – by mathematicians, at least – as continuous.

A **constant** number or quantity is one that does not change. *In the experiment we varied* [changed] *the amount of water in the beaker but kept the amount of salt added constant.*
A **random number** is one chosen by chance, i.e. it is not predictable.

B Working with numbers

The word **figure** is often used to refer to the symbol used for a number. *Write the total number in words and figures.*
Verbs that are frequently used with the word **number** include **calculate** [work out] a number, **estimate**[1] **a number, round a number up/down**[2], **total** [add up] a set of numbers. Numbers can also **tally**[3]. *My figures don't seem to tally with yours.* You can also **deduct** [take away, **subtract**] one number from another number.

[1] make a rough guess at [2] make a **fraction**, e.g. $\frac{1}{8}$ or 0.78 into the nearest **whole number**
[3] match, agree

Values and **variables** are also useful terms when working with numbers. **Values** are individual numbers in a set of data. *The graph shows the temperature **values** for different months of the year.* **Variables** are characteristics that can take on different values for different members of a group or set being studied. *In investigating living standards you must take key **variables** such as social provision and cost of living into account.*

The **incidence** of something refers to how frequently it occurs. *The **incidence** of twins in the population is growing.* When talking about numbers, **magnitude** simply refers to the size of something, whereas in other contexts it indicates large size or importance. *Write down the numbers **in order of magnitude**, beginning with the smallest.*

When **making calculations** in, say, an exam, it is often a good idea to make an **estimate**[4] first of what the answer is likely to be. Then you will see if your final answer is **in the right area**[5] or not. Exam candidates are also often advised to **show their workings**[6] so that the marker can see how they **arrived at their answer** and they may get credit for their method even if the final answer is incorrect.

[4] rough guess [5] approximately the same [6] leave all their calculations on the page

Giving feedback

Giving praise and encouragement

Yes, that's correct. Well done.
Yes, that's much better.
Good.
Fantastic!
Great!
Super!
Splendid!
Brilliant!
Perfect!
Excellent!
Wow!

Dealing with wrong answers

I'm afraid that's not the right answer.
I'm afraid that's incorrect / not quite right.

Commenting on written homework

You did this really well.
You're making excellent progress. See if you can do even better next time.
I'm really impressed by your work.
I really like the work you've done.
This was a good piece of work. Well done.
I enjoyed reading this. It was very good.
This was an impressive piece of work.
Well done. You must have put a lot of time and effort into doing this.

Dealing with homework that is not up to standard

As a general principle it is best to state what you want in positive terms. In other words, say what is needed or what you want rather than criticise something for being unsatisfactory. Criticism is always backward looking, so say what you want in the future. For example, rather than say "This homework was poorly organised" it is better to say "Your homework needs to be better organised."
Here are some ways of phrasing things positively and being forward looking:

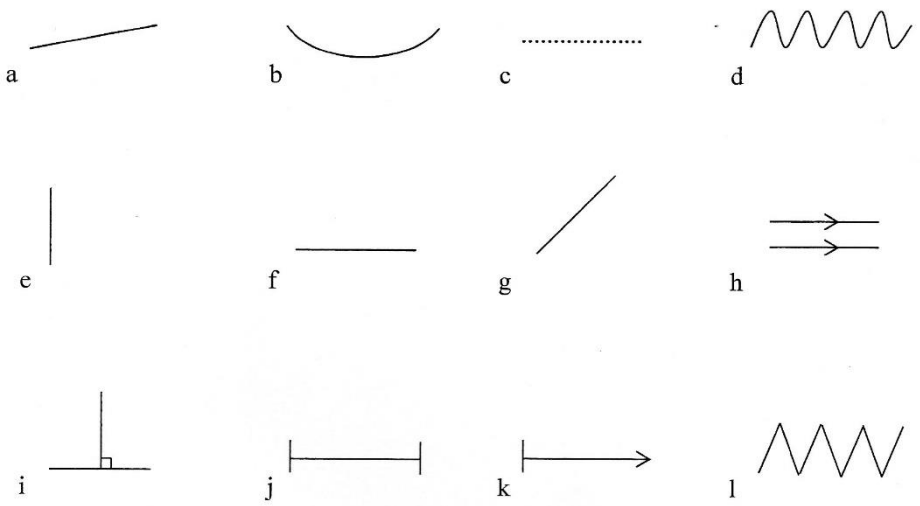
You need to improve ...

You need to work on ...

You need to pay more attention to ...	organisation layout presentation detail the use of section headings proof-reading your work before you hand it in.

I think there is still room for improvement in the areas of ...

Lines



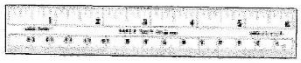
Exercise 1
Match the lines above with the words below and mark where the word is stressed.

f horizontal line	e vertical line	a straight line
d wavy line	i perpendicular lines	b curved line
l zigzag line	g diagonal line	c dotted line
h parallel lines	j line segment	k ray

Exercise 2

1 If you make a mistake, you can rub it out with a rubber.

2 If you want to draw and measure straight lines you need a ruler.



linear

3 If an object moves in a straight line, it has rectilinear motion.

4 orthogonal ≡ perpendicular.

5 A piece of straight line between two points is a line segment.

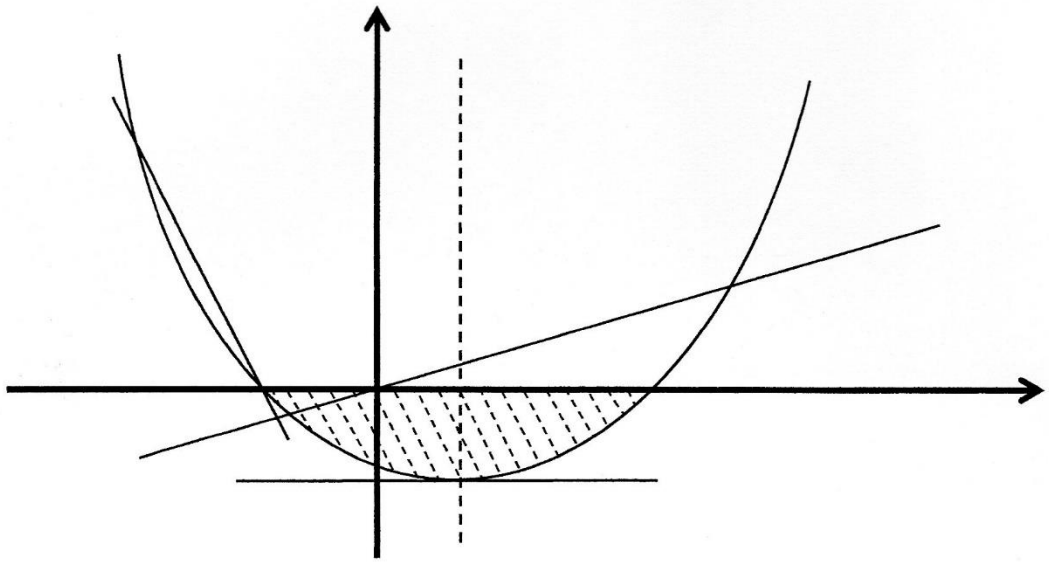
6 A straight line which has a starting point but no endpoint is a ray.

Talking about graphs

Exercise 1

Look at the words below. Listen to your teacher pronounce them and mark the stress and sounds of the words. Then **label** the diagram and describe it to your partner.

x-axis	horizontal	vertical	linear graph	above the *x*-axis
y-axis	parabola	to cross	quadratic graph	below the *x*-axis
axes	secant	tangent	point of intersection	shaded area
vertex	linear equation	*x*-intercept	point of inflection	origin
point	straight line	*y*-intercept	is perpendicular to	turning point
curve	axis of symmetry	curved line	inverted parabola	to intersect
Cartesian coordinates (*x*,*y*)		the parabola opens upwards/downwards/sideways		
positive/negative gradient (slope)		the steepness of the slope		1st, 2nd, 3rd, 4th quadrant



Exercise 2 What is the difference between *to plot a graph* and *to sketch a graph*?

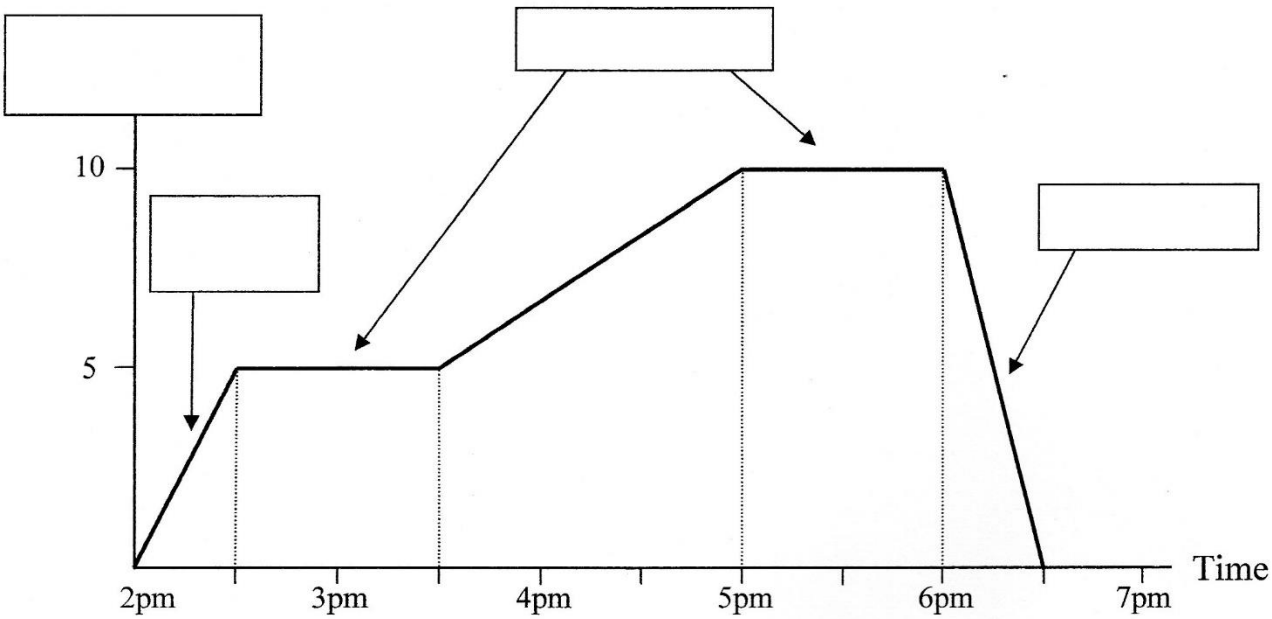
Exercise 3 What are the missing words?

1 Look at this point On the line.
2 The lines intersect at this point.
3 By looking at the graph we can see that the speed increases with time.
4 This line passes through the origin.
5 Plot the graph of voltage (*y*-axis) against current (*x*-axis).
6 The y-intercept can be read off the graph.

Create your own graphs and look at mind maps for maths revision

You can create your own graphs at: www.createagraph.com
Mind maps for maths revision are at: http://www.mathsrevision.com

Travel graphs: worksheet



1 A travel graph is always _____.

2 The _____ of a line on the graph represents the _____ of an object, i.e. how fast it is moving, e.g. 70km/h (kilometres per hour).

3 The speed of an object = _____.

4 The _____ the graph, the greater the _____.

5 The horizontal lines on the graph are where the object is _____, i.e. it is _____.

6 The line representing the return journey _____.

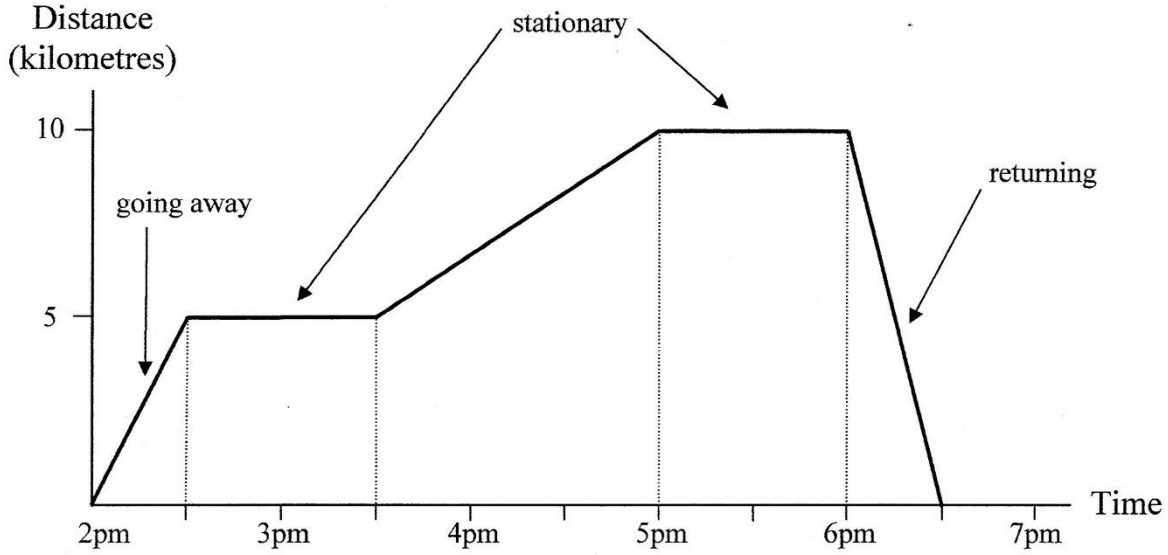
7 When an object is moving at a _____ speed, the line on the graph is straight, but sloped.

NB: The *speed* of an object is how fast it is moving, e.g. 40km/h, and is a _____ quantity, whereas the *velocity* of an object is its speed in a _____, and is a _____ quantity, e.g. 40km/h east.

Question
Now work with your partner. What is the speed of the return journey?

Answer
Speed = gradient = _____ = _____ metres/minute = ____ km/h.

Travel graphs: wall copy



1 A travel graph is always distance against time.

2 The gradient/slope of a line on the graph represents the speed of an object, i.e. how fast it is moving, e.g. 70km/h (kilometres per hour).

3 The speed of an object $= \dfrac{\text{change in the vertical axis (distance)}}{\text{change in the horizontal axis (time)}}$

4 The steeper the graph, the greater the speed of the object.

5 The horizontal lines on the graph are where the object is not moving, i.e. it is stationary.

6 The line representing the return journey slopes downwards.

7 When an object is moving at a constant speed, the line on the graph is straight, but sloped.

NB: The *speed* of an object is how fast it is moving, e.g. 40km/h, and is a scalar quantity, whereas the *velocity* of an object is its speed in a particular direction, and is a vector quantity, e.g. 40km/h east.

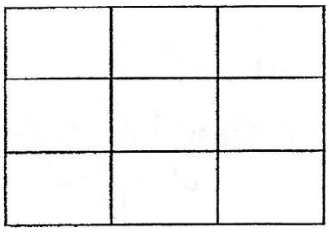
Question
Now work with your partner. What is the speed of the return journey?

Answer
Speed = gradient = ———— = _____ metres/minute = ____ km/h.

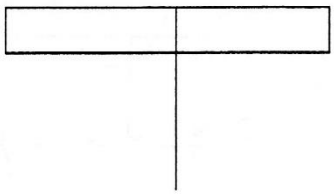
Visuals 2

Exercise 1 Match the words in the box with the visual organisers and diagrams below.

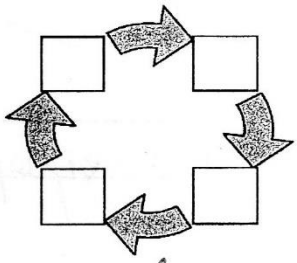
model	flow chart/diagram	mind map
table	grid	cycle
T-chart	cross-section	tree diagram



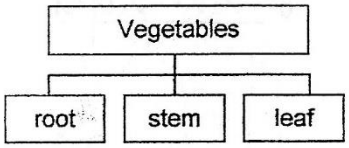
1 grid



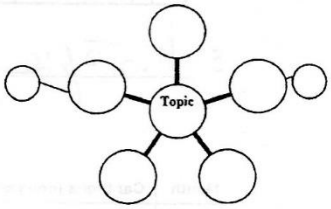
2 T-chart



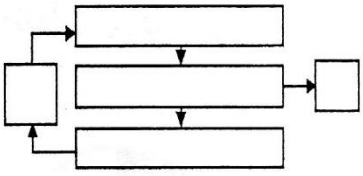
3 cycle



4 tree diagram



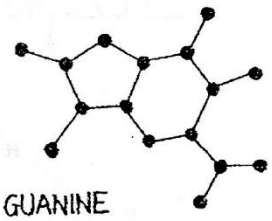
5 mind map



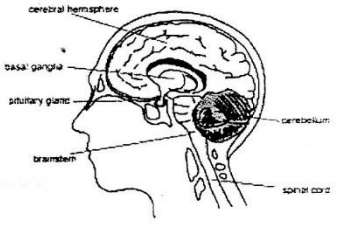
6 flow chart

	J	F	M	A	M
A	0.1	0.4	0.1	0.2	0.1
B	0.2	0.6	0.1	0.2	0.2
C	0.2	0.5	0.2	0.2	0.1
D	0.3	0.6	0.6	0.1	0.1

7 Table



8 model



9 ~~cerebrum~~ cross-section of the brain

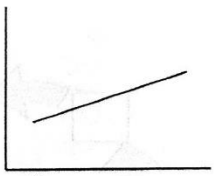
Exercise 2 Discuss with your partner what these visuals are used for.

Visuals 1

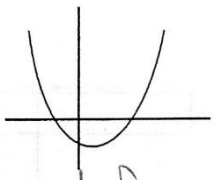
cumulative frequency diagram

Exercise 1 Match the words in the box with the graphs and visual organisers below.

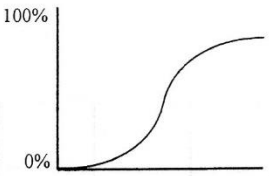
pie chart	Carroll diagram	tree diagram	line graph
pictogram	cumulative frequency diagram	linear graph	bar chart
histogram	quadratic graph	scatter diagram	Venn diagram



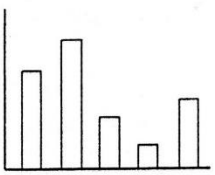
1 line graph



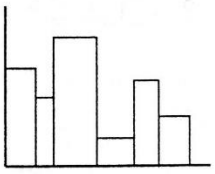
2 quadratic graph



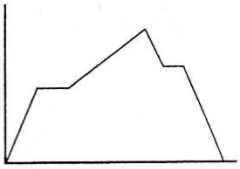
3 cumulative frequency diagram (ogive)



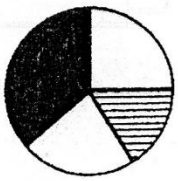
4 bar chart



5 histogram



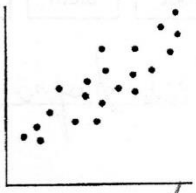
6 line graph



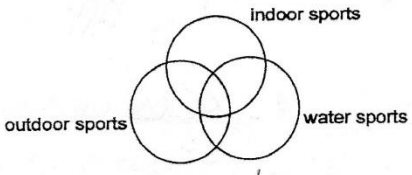
7 pie chart

Month	Car thefts (one car = 100 thefts)
Mar	
Apr	
May	
Jun	
Jul	
Aug	

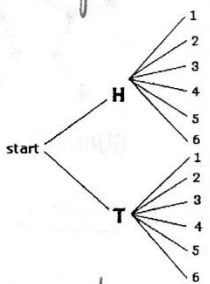
8 pictogram



9 scatter diagram



10 Venn diagram



11 Tree diagram

	Food	Drink
Natural	apples	water
Manufactured	pasta	coke

12 Carroll diagram

Exercise 2 Discuss with your partner what these visuals are used for.

Talking about visuals

Exercise 1 Fill in the missing words.

1 What are the implications ___of___ this?

2 Let's consider this ___in___ more detail.

3 What conclusions can we draw ___from___ this?

4 This line stands ___for___ the velocity of the object.

5 Let's look ___at___ this part of the graph.

6 I'd like to draw your attention ___to___ the shaded area under the curve.

7 I'd like us to focus ___on___ this point here.

8 What is the significance ___of___ this?

questi parole vanno sempre insieme

Language for talking about visuals

Practise saying the phrases below with the correct stress and intonation.

What this visual (chart, graph, etc.) shows is .....

This bar chart shows the distribution of .....

This line stands for (represents) .....

The characteristic features of ..... are .....

As you can see, ...

Let's have a look at .....

Let's take a closer look at .....

I'd like you to focus on this point here.

Let's look at this in more detail.

I'd like to draw your attention to .....

Here we can see .....

What are the implications of this?

What is the significance of this?

What conclusions can we draw from looking at this graph ?

Describing change

to fall, to decrease, to drop, to decline, to go down
to show a gradual decrease in, to show a gradual decline in

Adverbs: gradually / slowly / slightly / steadily

to fall, to decrease, to drop, to decline, to go down,
to show a rapid decrease in, to show a sudden decline in

Adverbs: sharply / steeply / rapidly / suddenly / dramatically

to rise, to increase, to go up, to climb, to show a gradual increase in

Adverbs: gradually / slowly / slightly / steadily

to rise, to increase, to go up, to shoot up, to show a rapid increase in

Adverbs: sharply / steeply / rapidly / suddenly / dramatically / exponentially

to peak, to reach a peak / a maximum / a high point

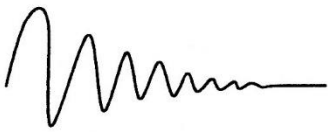
to reach a minimum / a low point

to dip, to be a dip in something

to remain stable, to remain constant, to stay at the same level, to stay the same

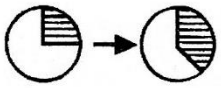
to flatten out, to level off, to reach a plateau /ˈplætəʊ/

to fluctuate /ˈflʌktʃueɪt/, to vary /ˈveəri/

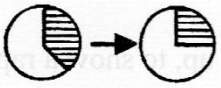


to stabilise /ˈsteɪbəlaɪz/

a steady rise, a continuous rise, a monotonic increase (i.e. it never decreases)



to expand, to grow



to decline, to shrink

Pie charts are often used in statistics and business.
Note: we talk about the *segment* of a pie chart.

Talking about figures and results

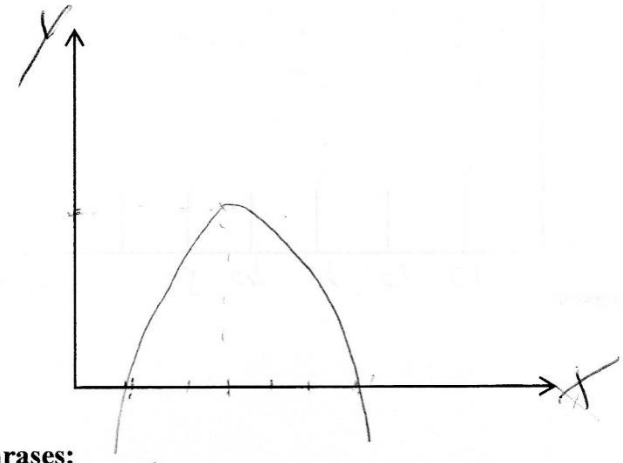
The result/figure is	under ...	just under ...	well under ...
	above ...	just above ...	well above ...
	roughly ...		
	approximately ...		
	more or less ...		
	in the region of ...		

Note: *just under* = a little under, *well under* = a lot under
just above = a little above, *well above* = a lot above
e.g. Her IQ is well above the national average.

Prepositions
to increase/rise/fall/drop *by* 10%
an increase/rise/fall/drop *of* 10% (a numerical amount)
an increase/rise/fall/drop *in* pressure (a noun)

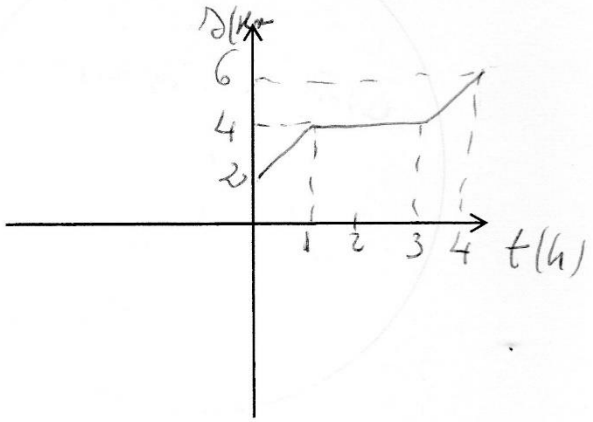
Talking about visuals and graphs

Choose one or more of the visuals below and draw something related to a specific topic. Make notes on the language you will need to talk about it. Then describe to your partner what it represents.



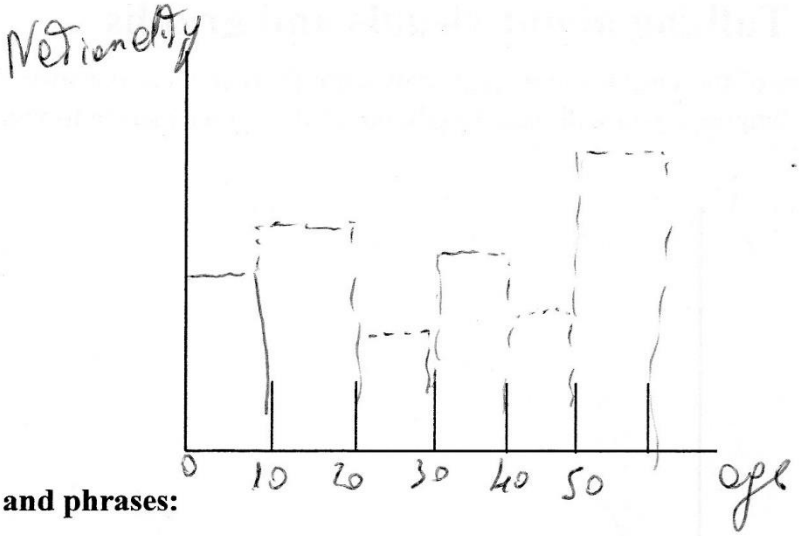
Key words and phrases:

In this graph describe the summit and the point of intersection with x-axes

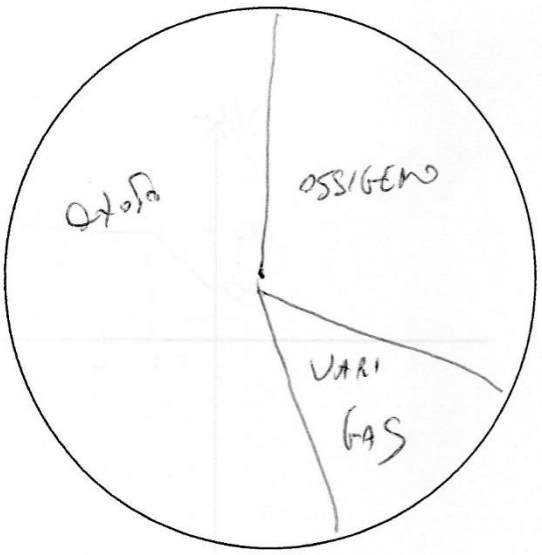


Key words and phrases: increase, stop the same, or stops for 2 hours

PTO



Key words and phrases:



Key words and phrases: